Name : .....

Fifth Semester B.Tech. Degree Examination, December 2015
(2013 Scheme)
13.501 : ENGINEERING MATHEMATICS – IV (AFRT)
(Complex Analysis and Linear Algebra)

Time : 3 Hours Max. Marks : 100

PART – A

Answer **all** question, **each** question carries **4** marks.

1. Show that the function $|z|^2$ is not regular at all points.

2. Evaluate $\oint_C \frac{3z^2+z}{z-1}dz$

   where C is the circle $|z+1|=1$.

3. Find the singular points of $\frac{z^2-2z}{(z+1)^2(z^2+4)}$.

4. Express (2, 5, –4, 0) as a linear combination of (2, –2, –5, 4), (1, 3, 2, 1) and (2, –1, 3, 6).

5. Find two unit vectors orthogonal to (1, –2, 3) and (1, 2, 3).

PART – B

Answer **one full** question from **each** Module, **each** question carries **20** marks.

Module – 1

6. a) If f(z) is a regular function of z, prove that

$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2.$$

P.T.O.

b) Determine the analytic function whose imaginary part is $\frac{x-y}{x^2+y^2}$.

c) Prove that $U = x^2 - y^2 - 2xy - 2x + 3y$ is harmonic. Find a function V such that $f(z) = U + iV$ is analytic. Also Express f(z) in terms of z.

7. a) If $f(z) = U + iV$ is an analytic function, find f(z) if $U - V = e^x(\cos y - \sin y)$.

b) Find the image of the following curves under the mapping $z = \frac{1}{z}$

i) The line $y - x + 1 = 0$

ii) The circle $|z - 3| = 5$.

c) Find the bilinear transformation which maps the points $z = 1, i, -1$ into the points $w = i, 0, -i$ and find the image of the line $|z| < 1$ under this transformation.

Module – 2

8. a) Evaluate $\oint_C \frac{dz}{\sinh z}$. Where C is the circle $|z| = 4$.

b) Determine the residue at the poles of $\frac{z^2 - 2z + 1}{(z+1)^3 (z^2+4)}$.

c) Expand $f(z) = \frac{(z-2)(z+2)}{(z+1)(z+4)}$

in the region

i) $|z| < 1$

ii) $1 < |z| < 4$

iii) $|z| > 4$.

9. a) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta d\theta}{5+4\cos\theta}$.

   b) Evaluate $\int_{-\infty}^{\infty} \frac{x^2-x+2}{(x^4+10x^2+9)} dx$.

Module – 3

10. a) Is the set S = {(2, 1, –2), (–2, –1, 2), (4, 2, –4)} for a basis for R^3, why ?

    b) Find the transition matrix from the basis $v_1 = (1, 2, 3)$, $v_2 = (1, 0, 1)$, $v_3 = (1, 2, 1)$ to the basis $u_1 = (1, 1, 0)$, $u_2 = (0, 1, 1)$, $u_3 = (1, 1, 1)$.

11. a) Find the best least squares fit by a linear function to the data

x	–1	0	1	2
y	0	1	3	9

    b) Show that the mapping $T : R^3 \to R^3$ is defined by

    $T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$

    is linear. Find a basis for the Kernel of T.

Module – 4

12. a) Find an orthonormal basis for the subspace of R^4 spanned by

    $(1, 1, 1, 1)^T$, $(2, 3, 2, -4)^T$ and $(-1, 5, -2, -1)^T$.

    b) Find a maxima or minima of $5x^2 + 5y^2 - 4xy$ subject to the constraint $XX^T = 1$ where $X = \begin{bmatrix} x \\ y \end{bmatrix}$.

13. a) Find a Singular value decomposition of $A = \begin{pmatrix} 1 & -1 \\ -2 & 2 \\ 2 & -2 \end{pmatrix}$.

    b) Reduce $8x^2 + 7y^2 + 3z^2 - 12xy + 4xz - 8yz$ into canonical form by orthogonal transformation and examine the definiteness.